



**GCE A LEVEL**

**1300U30-1**

**WEDNESDAY, 4 JUNE 2025 – MORNING**

**MATHEMATICS – A2 unit 3**

**PURE MATHEMATICS B**

**2 hours 30 minutes plus your additional  
time allowance**

**Surname** \_\_\_\_\_

**First name(s)** \_\_\_\_\_

**Centre Number** \_\_\_\_\_

**Candidate Number** 2 \_\_\_\_\_

| For Examiner's use only |              |              |
|-------------------------|--------------|--------------|
| Question                | Maximum Mark | Mark Awarded |
| 1                       | 11           |              |
| 2                       | 12           |              |
| 3                       | 6            |              |
| 4                       | 6            |              |
| 5                       | 8            |              |
| 6                       | 8            |              |
| 7                       | 5            |              |
| 8                       | 5            |              |
| 9                       | 4            |              |
| 10                      | 3            |              |
| 11                      | 10           |              |
| 12                      | 7            |              |
| 13                      | 11           |              |
| 14                      | 11           |              |
| 15                      | 7            |              |
| 16                      | 6            |              |
| Total                   | 120          |              |

**ADDITIONAL MATERIALS**

**In addition to this examination paper, you will need:**

- **a Formula Booklet;**
- **a calculator.**

**INSTRUCTIONS TO CANDIDATES**

**Use black ink, black ball-point pen or your usual method.**

**You may use a pencil for graphs and diagrams only.**

**Write your name, centre number and candidate number in the spaces provided on the front cover.**

**Answer ALL questions.**

**(Turn over)**

**Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.**

## **INFORMATION FOR CANDIDATES**

**The maximum mark for this paper is 120.**

**The number of marks is given in brackets at the end of each question or part-question.**

**Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.**

**Answers without working may not gain full credit.**

**(Turn over)**

**4**

**Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.**

**You are reminded of the necessity for good English and orderly presentation in your answers.**

**(Turn over)**

**REMINDER: Sufficient working must be shown to demonstrate the MATHEMATICAL method employed.**

**1 (a) Differentiate each of the following functions with respect to  $X$**

**(i)  $(e^{-x} + x^2)^3$  [2 marks]**

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$$e^{2x} \sin 3x$$

**[3 marks]**

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1 (a) (iii)

$$\frac{\cos 2x}{x^2 + 1}$$

[3 marks]

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(Turn over)

**10**

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**(Turn over)**

1 (b) A function is defined implicitly by

$$2x - 3x^2y^3 = 5$$

Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$

[3 marks]

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**2 (a) Find all values of  $x$  in the range  $0 \leq x \leq 2\pi$  satisfying the equation**

$$\cos^2 x + \sin x = 0$$

**Give your answers in terms of  $\pi$**   
**[6 marks]**

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**(Turn over)**

14

[illegible]

**(Turn over)**

[illegible]

**(Turn over)**

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**2 (b) Find all values of  $\theta$  in the range  $0^\circ \leq \theta \leq 360^\circ$  satisfying the equation**

$$2\sec^2 \theta - 5 = \tan \theta \quad [6 \text{ marks}]$$

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3. Given that  $\theta$  is small, find an estimate for the solution of the equation

$$\frac{\cos \theta}{1 + \sin 2\theta} = 0.8$$

[6 marks]

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- 4 (a) Using the trapezium rule with 5 ordinates, find an approximate value for

$$\int_0^{\frac{\pi}{3}} \sec^2 x dx$$

Show your working and give your answer correct to three decimal places. [4 marks]

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4 (b) Hence, deduce an approximate value for

$$\int_0^{\frac{\pi}{3}} (\tan^2 x + 2) dx \quad [2 \text{ marks}]$$

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5. A curve is defined by

$$f(x) = x^3 - 6x^2 + 12x - 5$$

- (a) Find the coordinates of the stationary point and determine its nature. [6 marks]

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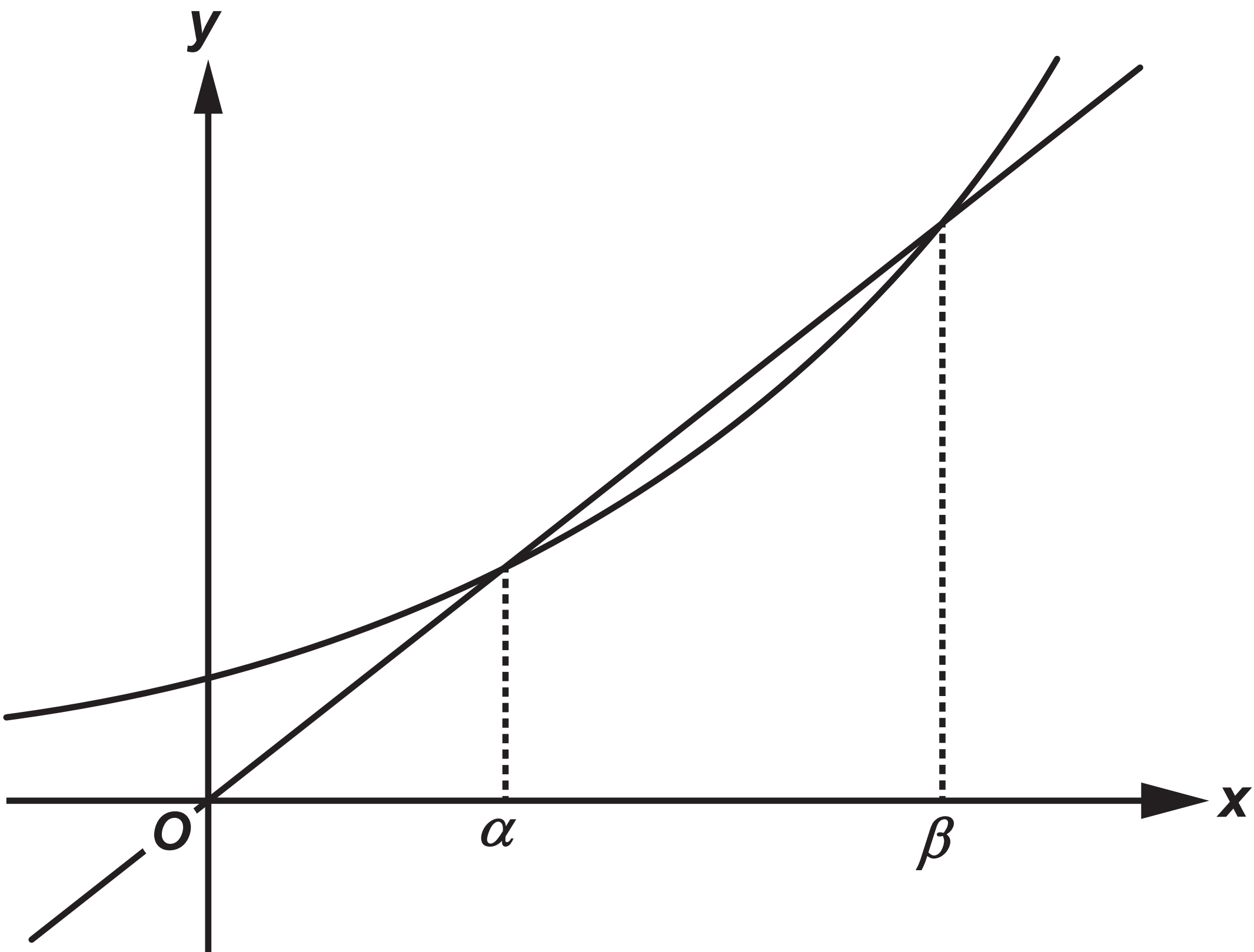
**5 (b) Determine the range of values of  $X$  where the curve is convex. [2 marks]**

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6. The diagram opposite shows a sketch of the curve with equation  $y = e^x$  and the straight line with equation  $y = 3x$

The straight line intersects the curve at the points where  $x = \alpha$  and  $x = \beta$

Approximate values for  $\alpha$  and  $\beta$  are  $0.62$  and  $1.5$  respectively.

6 (a) The iterative formula  $x_{n+1} = \frac{1}{3}e^{x_n}$  is used to find a more accurate approximation for  $\alpha$

Taking  $x_0 = 0.62$ , use the iterative formula to obtain the values of  $x_1$ ,  $x_2$  and  $x_3$

Give your answers correct to three decimal places. [2 marks]

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**6 (b) The Newton-Raphson method is used to find a more accurate approximation for  $\beta$**

**(i) Taking  $f(x) = e^x - 3x$  and starting with  $x_0 = 1.5$ , apply the Newton-Raphson method once to obtain a second approximation,  $x_1$**

**Give your answer correct to three decimal places. [3 marks]**

**(ii) Show that  $x_1$  is the value of  $\beta$  correct to three decimal places.**

**[2 marks]**

**(Turn over)**

**6 (b)(iii)**

**Explain why the Newton-Raphson method fails if the starting value in the approximation of  $\beta$  is  $\ln 3$**

**[1 mark]**

[illegible]

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- 7. The first and last terms of an arithmetic series are  $-3$  and  $61$  respectively. The sum of all of the terms is  $957$**

**Determine**

- (a) the number of terms, [2 marks]**

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50

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**7 (b) the common difference, [2 marks]**

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**7 (c) the middle term.**

**[1 mark]**

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8. By algebraic division, or otherwise, simplify the expression

$$\frac{x^4 + 5x^3 - 7x^2 - 29x + 30}{3(x + 5)(x + 3)(x - 2)}$$

[5 marks]

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**9. Use proof by contradiction to show that there are no integers  $x$  and  $y$  for which  $3x + 15y = 2$**

**[4 marks]**

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10 (a)

Sketch the graph of  $y = \sec x$  for

$$0 \leq x \leq \alpha, \text{ where } 0 < \alpha < \frac{\pi}{2}$$

[1 mark]

(Turn over)

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10 (b)

Explain why  $\int_0^{\alpha} \sec x \, dx > \alpha$   
[2 marks]

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11. Find  $\int_1^2 \frac{5x + 2}{(x + 1)^2 (2x - 1)} dx$

giving your answer in the form  
 $\ln a + b$  where  $a, b$  are  
constants.

[10 marks]

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70

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**(Turn over)**

[illegible]

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12. Expand  $\frac{2-x}{\sqrt[3]{1-2x}}$  in ascending powers of  $x$  up to and including the term in  $x^3$

State the range of values of  $x$  for which the expansion is valid.

[7 marks]

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13. A curve is defined parametrically by

$$x = 2t^3 + 1$$

$$y = 3t^2$$

(a) Show that the gradient of the curve  
at the point with parameter  $p$  is  $\frac{1}{p}$

[4 marks]

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**13 (b)**

**Find the equation of the normal to the curve at the point with parameter  $t$ , where  $t = 1$  [4 marks]**

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**13 (c) Determine the equation of the tangent to the curve at the point with parameter  $t$ , where  $t = 0$**

**[3 marks]**

[illegible]

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14 (a)

Use integration by parts to find the

EXACT value of  $\int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$

[6 marks]

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14 (b)

Use the substitution  $u = 1 + \ln x$ 

to find  $\int \frac{\ln x}{x(1 + \ln x)^2} dx$

[5 marks]

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15. A function  $f$  is defined by

$$f(x) = \frac{9}{(x-3)^2} \text{ where } x > 3$$

(a) Determine the range of  $f$

[2 marks]

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**(Turn over)**

**15 (b)**

**Find an expression for  $f^{-1}(x)$**

**[5 marks]**

[illegible]

**(Turn over)**

100

**(Turn over)**



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- 16. A person borrows £50 000 which is to be repaid over 20 years in 20 equal annual amounts. The lender charges an annual compound interest rate of 5% which remains constant throughout the period of the loan. The interest is added at the end of each year.**

**Each instalment is paid AFTER the interest has been added to the account, with the first instalment paid one year after the loan was taken out.**

**Calculate the amount of each annual instalment. [6 marks]**

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