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Centre number

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Candidate signature

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I declare this is my own work.

# INTERNATIONAL AS FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Monday 8 January 2024      07:00 GMT      Time allowed: 1 hour 30 minutes

## Materials

- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

## Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

## Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

| For Examiner's Use |      |
|--------------------|------|
| Question           | Mark |
| 1                  |      |
| 2                  |      |
| 3                  |      |
| 4                  |      |
| 5                  |      |
| 6                  |      |
| 7                  |      |
| 8                  |      |
| 9                  |      |
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| 11                 |      |
| <b>TOTAL</b>       |      |



J A N 2 4 F M 0 2 0 1

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Answer **all** questions in the spaces provided.

$$\frac{dy}{dx} = \frac{1}{x^2 - y + 3}$$

**[5 marks]**

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Answer \_\_\_\_\_

5



**2** The matrix **A** is defined by  $\mathbf{A} = \begin{bmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{bmatrix}$

The matrix **B** represents a reflection in the line  $y = \left(\tan \frac{\pi}{3}\right)x$

**2 (a)** Write down the matrix **B**

Give each element of the matrix in exact form.

**[1 mark]**

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Answer \_\_\_\_\_

**2 (b) (i)** Find the matrix **BA**

Give each element of the matrix to four decimal places.

**[2 marks]**

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Answer \_\_\_\_\_



**2 (b) (ii)** Describe fully the transformation represented by the matrix **BA**

**[3 marks]**

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**6**

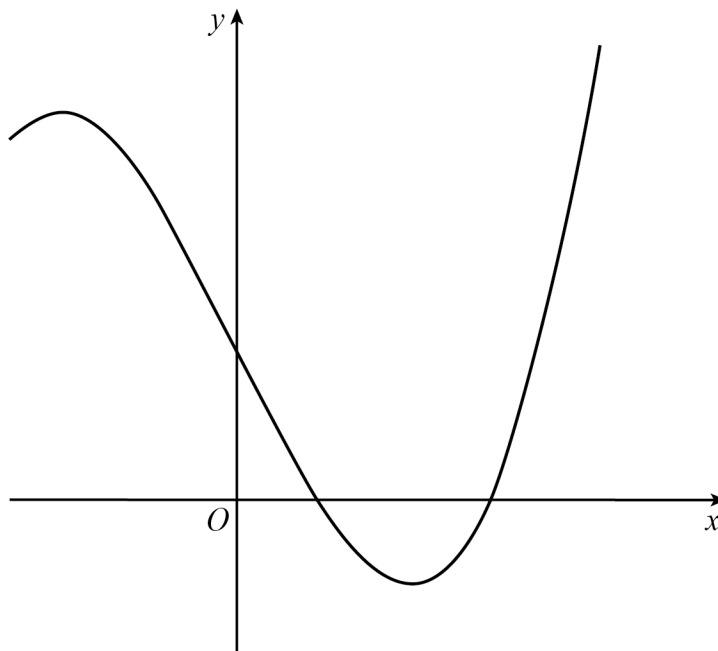
**Turn over for the next question**

**Turn over ►**



3

The graph shows part of the curve  $y = f(x)$  where  $f(x) = 1 - 2x + \frac{1}{2}x^3$



The equation  $f(x) = 0$  has two positive roots  $\alpha$  and  $\beta$

It is given that  $\alpha < 1$  and that  $\beta > 1$

3 (a) Show that  $\frac{3}{2} < \beta < \frac{7}{4}$

[2 marks]

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use linear interpolation once to show that  $\frac{209}{126} < \beta < \frac{7}{4}$

[illegible]

7

- 4 The variables  $P$  and  $Q$  are believed to be related by the equation

$$Q = aP^b$$

where  $a$  and  $b$  are constants.

The table below shows some values of  $P$  and  $Q$  obtained in an experiment.

|     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|
| $P$ | 5   | 7   | 10  | 15  | 20  | 30  |
| $Q$ | 3.0 | 2.6 | 1.9 | 1.5 | 1.3 | 1.0 |

- 4 (a) Let  $X = \log_{10} P$  and  $Y = \log_{10} Q$

Complete the table below, giving values to two decimal places.

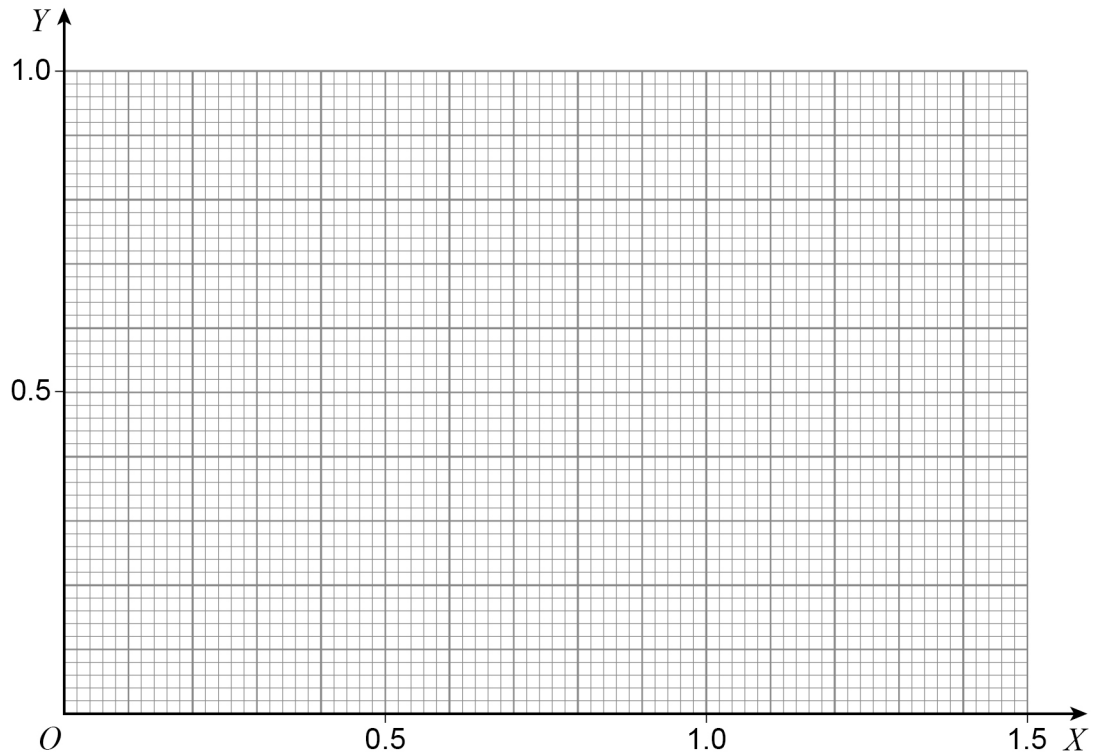
[2 marks]

|     |      |      |      |      |  |      |
|-----|------|------|------|------|--|------|
| $X$ | 0.70 | 0.85 | 1.00 |      |  |      |
| $Y$ | 0.48 | 0.41 |      | 0.18 |  | 0.00 |



- 4 (b)** Plot the values of  $X$  and  $Y$  on the grid below and draw a line of best fit.

**[2 marks]**



- 4 (c)** Use your line of best fit to estimate the value of  $a$  and the value of  $b$

**[4 marks]**

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$a =$  \_\_\_\_\_

$b =$  \_\_\_\_\_

**8**

**Turn over ►**



**5 (a)** Find the equations of the invariant lines of the transformation represented by **M** [6 marks]

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Answer \_\_\_\_\_

**5 (b)** Find the equation of the line of invariant points of the transformation represented by **M** **[2 marks]**

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Answer \_\_\_\_\_

- 5 (c)** The transformation represented by **M** is a combination of two transformations: a reflection in the line  $y = \frac{1}{2}x$  followed by a transformation represented by the matrix **N**

Find the matrix **N****[6 marks]**

Answer \_\_\_\_\_

14

**Turn over ►**

**Section B****Statistics**

Answer **all** questions in the spaces provided.

- 6** A bag contains 32 blue counters, 14 red counters and 17 white counters.  
A counter is randomly selected from the bag, its colour is recorded and it is **not** replaced.  
A second counter is randomly selected from the bag and its colour is recorded.

- 6 (a)** Draw a tree diagram to represent this information.

**[3 marks]**



- 6 (b)** Find the exact probability that the first counter is blue, given that the second counter is white.

**[3 marks]**

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Answer \_\_\_\_\_

6

**Turn over for the next question**

**Turn over ►**



**7** On a test, 7% of students gain an A grade.

A teacher selects students at random and notes whether they have gained an A grade.

The random variable  $N$  represents the number of students the teacher selects up to and including the first student who gained an A grade on the test.

**7 (a)** Find the probability that the fourth student the teacher selects is the first to have gained an A grade.

Give your answer to four decimal places.

**[1 mark]**

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Answer \_\_\_\_\_

**7 (b)** Find the probability that the first three students the teacher selects did **not** gain an A grade.

**[2 marks]**

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Answer \_\_\_\_\_



- 7 (c) Find  $G_N(t)$ , the probability generating function for  $N$

Give your answer in the form  $\frac{at}{1-bt}$  where  $a$  and  $b$  are positive constants.

[3 marks]

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Answer \_\_\_\_\_

- 7 (d) The random variable  $M$  is such that the probability generating function

$$G_M(t) = 0.07t + 0.93$$

The random variables  $M$  and  $N$  are independent.

Find  $G_{M+N}(t)$ , the probability generating function for  $M + N$

Give your answer in the form  $\frac{xt^2 + yt}{1-zt}$  where  $x$ ,  $y$  and  $z$  are positive constants.

[2 marks]

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Answer \_\_\_\_\_



- 8** The random variable  $X$  has a discrete uniform distribution and takes values  $1, 2, 3, \dots, n$   
It is given that  $P(X > 26) = 2P(X \leq 9)$

- 8 (a)** Find the value of  $n$

**[2 marks]**

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Answer \_\_\_\_\_



**[4 marks]**

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Answer \_\_\_\_\_

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**Section C****Mechanics**

Answer **all** questions in the spaces provided.

- 9 (a)** Given that  $\theta$  is an angle, use the cosine rule formula to show that  $\cos\theta$  is dimensionless.

**[2 marks]**

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**9 (b)**

$$U^2 = rg(1 - \cos\theta)$$

where

$U \text{ ms}^{-1}$  is a speed

$r$  metres is a radius

$g \text{ ms}^{-2}$  is the acceleration due to gravity

$\theta$  is an angle

Show that the formula is dimensionally consistent.

**[3 marks]**

[illegible]

5

**Turn over ►**



**10** A boat is moving in the sea.

The boat has constant velocity  $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ m s}^{-1}$  relative to the sea.

The sea water has constant velocity  $\begin{bmatrix} 2 \\ -1 \end{bmatrix} \text{ m s}^{-1}$

At time  $t = 0$  the boat has displacement  $\begin{bmatrix} -200 \\ -60 \end{bmatrix}$  metres from a lighthouse.

**10 (a)** Find the resultant velocity of the boat.

**[1 mark]**

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Answer \_\_\_\_\_

**10 (b)** Find the minimum distance between the boat and the lighthouse.

**[5 marks]**

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Answer \_\_\_\_\_

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**Turn over ►**



Two particles  $A$  and  $B$  are moving towards each other along a straight line on a smooth horizontal surface.

Particle  $A$  has mass  $4\text{ kg}$  and particle  $B$  has mass  $6\text{ kg}$

After the collision particle  $B$  has speed  $1.48 \text{ m s}^{-1}$

The coefficient of restitution between the two particles is  $e$

Find the value of  $e$

Fully justify your answer.

**[5 marks]**

[illegible]

- 11 (b)** Find the magnitude of the impulse exerted on  $A$  by  $B$  during the collision.

[2 marks]

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Answer \_\_\_\_\_

- 11 (c)** Assume that the particles exert a constant force of 672 newtons on each other during the collision.

Find the time that the particles are in contact with each other.

[2 marks]

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Answer \_\_\_\_\_

9

**END OF QUESTIONS**

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