

INTERNATIONAL AS FURTHER MATHEMATICS FM02

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$hf(-2,3) = 0.1 \times \frac{1}{(-2)^2 - 3 + 3}$ $= 0.025$ $y_2 = 3 + 0.025 = 3.025$ $y_3 = 3.025 + 0.1 \times \frac{1}{(-1.9)^2 - 3.025 + 3}$ $[= 3.052894003]$ 3.0529	M1 A1 M1 M1 A1	correct substitution into RHS of this expression PI 3 + their value of $hf(-2,3)$ Correct substitution using their x_2 and y_2 into second term here Correct answer given to 4 dp
	Question 1 Total	5	

Q	Answer	Marks	Comments
2(a)	$\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$	B1	
		1	

Q	Answer	Marks	Comments
2(b)(i)	$\mathbf{BA} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{bmatrix}$ $= \begin{bmatrix} 0.1196 & -0.9928 \\ 0.9928 & 0.1196 \end{bmatrix}$	M1 A1	Two correct elements of their B multiplied by A
		2	

Q	Answer	Marks	Comments
2(b)(ii)	Rotation about origin $\theta = \cos^{-1}(0.119615)$ $= 1.451$, anticlockwise	B1 M1 A1ft	oe Must be the only transformation mentioned oe AWRT 1.5 or 83°, sense must be clear. Allow +1.5 ft their BA , provided it is a valid rotation matrix
		3	

	Question 2 Total	6	
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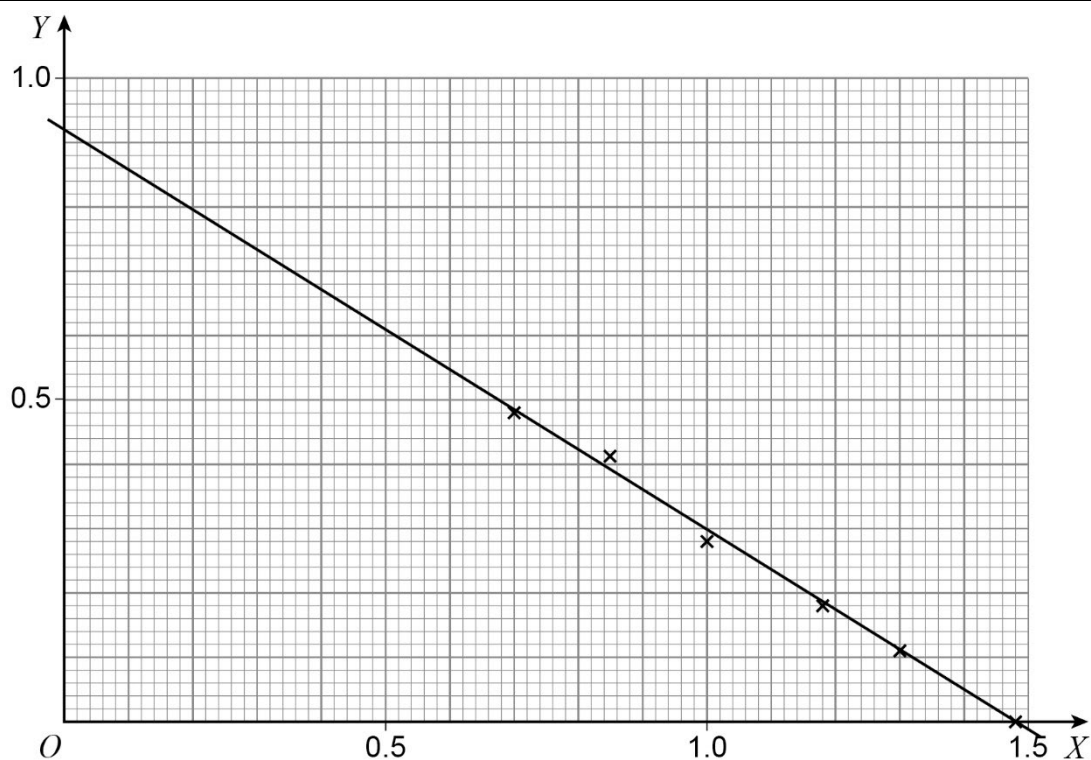
Q	Answer	Marks	Comments
3(a)	$f\left(\frac{3}{2}\right) = -\frac{5}{16} [= -0.3125]$ and $f\left(\frac{7}{4}\right) = \frac{23}{128} [= 0.1796875]$ sign change & continuous function, so the root β lies in the interval $\frac{3}{2} < x < \frac{7}{4}$	M1 A1	Correct evaluation of a suitable interval Must state that there is a change of sign and that the curve is continuous (condone unbroken) and concludes a root is present in the interval
		2	

Q	Answer	Marks	Comments
3(b)	$\frac{x_1 - \frac{3}{2}}{\frac{7}{4} - x_1} = \frac{\frac{5}{16}}{\frac{23}{128}}$ $\frac{23}{128}\left(x_1 - \frac{3}{2}\right) = \frac{5}{16}\left(\frac{7}{4} - x_1\right)$ $\frac{23}{128}x_1 - \frac{69}{256} = \frac{35}{64} - \frac{5}{16}x_1$ $\frac{63}{128}x_1 = \frac{209}{256}$ $x_1 = \frac{209}{126}$ $[f(x_1)] = -0.03556$ $f(x_1) < 0$ so $\frac{209}{126} < \beta < \frac{7}{4}$	M1 M1 A1 M1 A1	Obtains correct equation in terms of x_1 oe ft their values from (a) Obtains correct linear equation in terms of x_1 oe ft their values from (a) Must be convincingly shown For evaluating their $f(x_1)$ CSO
		5	

	Question 3 Total	7	
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Q	Answer					Marks	Comments
4(a)						M1 A1	M1: At least three correct entries; Condone 1.3 in place of 1.30 A1: All entries correct
	...	1.00	1.18	1.30	1.48		
	...	0.28	0.18	0.11	0.00		
						2	

Q	Answer	Marks	Comments
4(b)		B1ft	Points plotted ft their entries within half a square tolerance
		B1ft	Line of best fit seen



		2	
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Q	Answer	Marks	Comments
4(c)	$\log_{10} Q = \log_{10} a + b \log_{10} P$ $y\text{-intercept} = 0.92 \quad [= \log_{10} a]$ $\text{gradient} = -0.63 \quad [= b]$ $a = 10^{0.92} \quad \text{or} \quad b = -0.63$ $a = 8.3$ $b = -0.63$	M1 M1 M1 A1	Takes logs of both sides to reduce to linear form PI Sight of correct y -intercept or gradient value for their line of best fit Sets $a = 10^{\text{their } y\text{-intercept}}$ or $b = \text{their gradient}$ PI AWFW [7.8, 9.0] for a AWFW [-0.68, -0.58] for b
		4	
	Question 4 Total	8	

Q	Answer	Marks	Comments
5(a)	$[x' =] 0.4x + 1.2(mx + c)$ $[y' =] 1.2x - 1.4(mx + c)$	M1	Finds correct expressions for x' and y'
	$1.2x - 1.4(mx + c)$ $= m(0.4x + 1.2(mx + c)) + c$	M1	Sets their $y' = m(\text{their } x') + c$
	$1.2m^2 + 1.8m - 1.2 = 0$ $-1.4c - 1.2mc - c = 0$	m1	Attempt to find m or c by comparing coefficients
	$m = \frac{1}{2}, m = -2$	A1	Correct values of m
	lines are $y = -2x + c$ [where c is real]	A1	No restrictions on c
	$y = \frac{1}{2}x$	B1	
		6	

Q	Answer	Marks	Comments
5(b)	$0.4x + 1.2y = x$ and $1.2x - 1.4y = y$	M1	Or uses 5(a) and demonstrates that
	$\Rightarrow y = \frac{1}{2}x$	A1	$\begin{bmatrix} 0.4 & 1.2 \\ 1.2 & -1.4 \end{bmatrix} \begin{bmatrix} x \\ 0.5x \end{bmatrix} = \begin{bmatrix} x \\ 0.5x \end{bmatrix}$ PI by correct solution
		2	

Q	Answer	Marks	Comments
5(c)	$\tan \theta = \frac{1}{2} [\Rightarrow \theta = 0.4636]$ $\Rightarrow \cos 2\theta = 0.6 \text{ and } \sin 2\theta = 0.8$ $\begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$ so $\mathbf{M} = \mathbf{N} \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$ $\begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}^{-1} = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$ $\mathbf{N} = \begin{bmatrix} 0.4 & 1.2 \\ 1.2 & -1.4 \end{bmatrix} \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$ $\mathbf{N} = \begin{bmatrix} 1.2 & -0.4 \\ -0.4 & 1.8 \end{bmatrix}$	M1 A1 M1 B1ft M1 A1	Uses $\tan \theta = \frac{1}{2}$ to obtain values of $\cos 2\theta$ and $\sin 2\theta$ PI by correct matrix for reflection in $y = \frac{1}{2}x$ Obtains matrix for reflection in $y = \frac{1}{2}x$ Sets up correct equation for N PI by a correct expression for N later Clearly states that the reflection is self-inversing or find the correct inverse of their matrix for reflection in $y = \frac{1}{2}x$ PI Post-multiplies M by this inverse PI oe
		6	
	Question 5 Total	14	

Q	Answer	Marks	Comments
6(a)		M1 A1ft A1	Correct structure Probabilities on one set of branches correct or correct follow through from previous branches oe Fully correct oe
<pre> graph LR Root(()) --- 32/63 B1(B) Root --- 14/63 R1(R) Root --- 17/63 W1(W) B1 --- 31/62 B2(B) B1 --- 14/62 R2(R) B1 --- 17/62 W2(W) R1 --- 32/62 B3(B) R1 --- 13/62 R3(R) R1 --- 17/62 W3(W) W1 --- 32/62 B4(B) W1 --- 14/62 R4(R) W1 --- 16/62 W4(W) </pre>			
		3	

Q	Answer	Marks	Comments
6(b)	$\frac{\frac{32}{63} \times \frac{17}{62}}{\frac{32}{63} \times \frac{17}{62} + \frac{14}{63} \times \frac{17}{62} + \frac{17}{63} \times \frac{16}{62}}$ $= \frac{16}{31}$	M1 M1 A1	Correct numerator for their probabilities Sight of $\frac{272}{1953}$ Correct denominator for their probabilities oe eg $\frac{272}{1953} + \frac{17}{279} + \frac{136}{1953}$ or $\frac{17}{63}$ oe
		3	
	Question 6 Total	6	

Q	Answer	Marks	Comments
7(a)	$(1 - 0.07)^3 \times 0.07$ $= 0.0563$	B1	AWRT 0.0563
		1	

Q	Answer	Marks	Comments
7(b)	$(1 - 0.07)^3$ $= 0.804357$	M1 A1	Attempts correct calculation for either $P(N > 3)$ or $P(N \geq 3)$ AWRT 0.8044
		2	

Q	Answer	Marks	Comments
7(c)	$G_N(t) = \sum_{n=1}^{\infty} t^n \times 0.07 \times 0.93^{n-1}$ $= 0.07t \times (1 + 0.93t + 0.93^2 t^2 + \dots)$ $= \frac{0.07t}{1 - 0.93t}$	M1 M1 A1	Correct expression for $G_N(t)$ If written term by term, minimum that must be seen is $G_N(t) = 0.07t + 0.07(1 - 0.07)t^2 + 0.07(1 - 0.07)^2 t^3 + \dots$ Allow use of p for 0.07 PI Identify an infinite geometric series by either identifying kt ($k \neq 0$) as a factor or substituting their a and r into $\frac{a}{1 - r}$ PI oe
		3	

Q	Answer	Marks	Comments
7(d)	$G_{M+N}(t) = \frac{0.07t}{1-0.93t} \times (0.07t + 0.93)$	M1	Applies $G_{M+N}(t) = G_M(t) \times G_N(t)$ with their $G_N(t)$
	$G_{M+N}(t) = \frac{0.0049t^2 + 0.0651t}{1-0.93t}$	A1	oe
		2	

	Question 7 Total	8	
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Q	Answer	Marks	Comments
8(a)	$\frac{n-26}{n} = 2 \times \frac{9}{n}$	M1	Forms correct equation
	$n = 44$	A1	If M0 awarded, SC1 for final answer of $n = 43$
		2	

Q	Answer	Marks	Comments
8(b)	$\text{Var}(X) = \frac{44^2 - 1}{12} = 161.25$	B1ft	oe ft their n PI
	$\text{Var}(Y) = 34 \times 0.73(1 - 0.73) = 6.7014$	B1	PI
	$\text{Var}(4X - 10Y + 8)$	M1	Applies correct formula for $\text{Var}(4X - 10Y + 8)$ to their $\text{Var}(X)$ and $\text{Var}(Y)$
	$= 4^2 \text{Var}(X) + 10^2 \text{Var}(Y)$		
	$= 4^2 \times 161.25 + 10^2 \times 6.7014$	A1	oe
	$\text{Var}(4X - 10Y + 8) = 3250.14$	4	

	Question 8 Total	6	
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Q	Answer	Marks	Comments
9(a)	$\cos \theta = \frac{a^2 + b^2 - c^2}{2ab}$ $[\cos \theta] = \frac{[a^2 + b^2 - c^2]}{[2ab]} = \frac{L^2}{L^2} = L^0 = 1$ Therefore $\cos \theta$ is dimensionless	M1 A1	Applies dimensional analysis to the cosine formula and finds either $[a^2 + b^2 - c^2] = L^2$ or $[2ab] = L^2$ Condone $L \times L$ Finds $[a^2 + b^2 - c^2] = L^2$ and $[2ab] = L^2$ and concludes that $\cos \theta$ is dimensionless
		2	

Q	Answer	Marks	Comments
9(b)	$[U^2] = L^2 T^{-2}$ $[rg(1 - \cos \theta)] = [r][g][(1 - \cos \theta)]$ $= L \times L T^{-2} \times 1$ $= L^2 T^{-2}$ Therefore dimensionally consistent	M1 M1 A1	Finds dimensions of LHS Condone use of units Finds dimensions of RHS Condone use of units Obtains correct dimensions of both sides and concludes dimensionally consistent
		3	

	Question 9 Total	5	
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Q	Answer	Marks	Comments
10(a)	Resultant velocity = $\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $= \begin{bmatrix} 3 \\ 1 \end{bmatrix} \text{ [m s}^{-1}\text{]}$	B1	Correct resultant velocity Do not ignore subsequent working
		1	

Q	Answer	Marks	Comments
10(b)	$\mathbf{r} = \begin{bmatrix} 3t - 200 \\ t - 60 \end{bmatrix}$ $s^2 = (3t - 200)^2 + (t - 60)^2$ $= 10t^2 - 1320t + 43600$ $\frac{ds^2}{dt} = 20t - 1320 = 0$ $t = \frac{1320}{20} = 66$ $s^2 = (3 \times 66 - 200)^2 + (66 - 60)^2$ $= 40$ $s = 2\sqrt{10} \text{ [metres]}$	M1 M1 M1 A1 A1	Finds position of boat relative to lighthouse ft their resultant velocity Finds distance or distance ² between the boat and the lighthouse ft their position of boat relative to lighthouse Uses a method to find the minimum distance (calculus or completing the square) for their quadratic PI by correct final answer Correct time or correct completion of the square $10(t - 66)^2 + 40$ PI by correct final answer Correct minimum distance AWRT 6.3
		5	

	Question 10 Total	6	
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Q	Answer	Marks	Comments
11(a)	There is a net momentum to the right, so at least B must move to the right after the collision $\therefore v_B = 1.48$ $4v_A + 6 \times 1.48 = 4 \times 5 - 6 \times 3$ $v_A = \frac{2 - 8.88}{4} = -1.72$ $1.48 - (-1.72) = -e(-3 - 5)$ $e = \frac{3.2}{8} = 0.4$	B1 M1 A1 m1 A1	Any equivalent correct statement M1: An equation for conservation of momentum using one of $v_B = -1.48$ or $v_B = 1.48$ A1: $v_A = -1.72$ or $v_A = 2.72$ oe Correct equation for coefficient of restitution ft their v_A and using one of $v_B = -1.48$ or $v_B = 1.48$ Correct coefficient of restitution
		5	

Q	Answer	Marks	Comments
11(b)	$I = 4 \times (-1.72) - 4 \times 5$ $= 26.88 \text{ [N s]}$	M1 A1	Uses formula for impulse Condone sign errors Correct magnitude. Must be positive AWRT 27
		2	

Q	Answer	Marks	Comments
11(c)	$26.88 = 672t$ $t = \frac{26.88}{672} = 0.04 \text{ [seconds]}$	M1 A1ft	Uses $I = Ft$ Correct time for their impulse
		2	

	Question 11 Total	9	
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