

INTERNATIONAL A-LEVEL FURTHER MATHEMATICS FM05

(9665/FM05) Unit FM2 Mechanics

Mark scheme

January 2024

Version: 1.1 Final



2 4 1 X F M 0 5 / M S

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$2 \begin{bmatrix} 4 \\ 7 \end{bmatrix} + 6 \begin{bmatrix} 6 \\ -4 \end{bmatrix} = 2 \begin{bmatrix} 7 \\ 1 \end{bmatrix} + 6\mathbf{v}_B$ $6\mathbf{v}_B = \begin{bmatrix} 30 \\ -12 \end{bmatrix}$ $\mathbf{v}_B = \begin{bmatrix} 5 \\ -2 \end{bmatrix} \text{ [m s}^{-1}\text{]}$	M1 A1 m1 A1	Applies conservation of momentum in a vector form. Correct vector equation Solves their vector equation for velocity of B Correct velocity
		4	

Q	Answer	Marks	Comments
1(b)	$\mathbf{I} = 2 \begin{bmatrix} 7 \\ 1 \end{bmatrix} - 2 \begin{bmatrix} 4 \\ 7 \end{bmatrix}$ $= \begin{bmatrix} 6 \\ -12 \end{bmatrix}$ $I = \sqrt{6^2 + 12^2}$ $= 6\sqrt{5} \text{ [N s]}$	M1 A1 M1 A1	Uses impulse equation with vectors Correct impulse Finds magnitude of their impulse Correct magnitude of the impulse
		4	

	Question 1 Total	8	
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Q	Answer	Marks	Comments
2(a)	$4 = \frac{2\pi}{\omega}$ $\omega = \frac{\pi}{2}$	M1	Correct value of ω
	$v_{\max} = \frac{3}{4} \times \frac{\pi}{2} = \frac{3\pi}{8} \text{ [m s}^{-1}\text{]}$	A1	Correct speed oe in terms of π (eg 0.375π)
		2	

Q	Answer	Marks	Comments
2(b)	$v^2 = \omega^2(a^2 - x^2)$ $v^2 = \left(\frac{\pi}{2}\right)^2(0.75^2 - 0.45^2)$	M1	Uses SHM speed formula or other SHM method with their amplitude and consistent displacement.
	$v = \frac{3\pi}{10} \text{ [m s}^{-1}\text{]}$	A1ft	Correct speed for their amplitude. oe in terms of π (eg 0.3π)
		2	

Q	Answer	Marks	Comments
2(c)	$x = \frac{3}{4} \cos\left(\frac{\pi t}{2}\right)$	M1	Finds expression for displacement in terms of time
	$0.45 = \frac{3}{4} \cos\left(\frac{\pi t}{2}\right)$	A1ft	Correct equation for time, for their amplitude.
	$t = \frac{2}{\pi} \cos^{-1}\left(\frac{3}{5}\right)$ $t = 0.5903 \text{ [seconds]}$	A1	Correct time
		3	

	Question 2 Total	7	
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Q	Answer	Marks	Comments
3(a)	$8\cos 60^\circ = v\cos 30^\circ$ $4 = \frac{\sqrt{3}}{2}v$ $v = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$	M1 A1	M1: Equation for motion parallel to wall with at least one side correct A1: Fully correct
		A1	Correct value for v ACF
		3	

Q	Answer	Marks	Comments
3(b)	$e \times 8\sin 60^\circ = v\sin 30^\circ$ $e \times 4\sqrt{3} = \frac{4\sqrt{3}}{3}$ $e = \frac{1}{3}$	M1 A1ft	M1: Equation for motion perpendicular to wall with at least one side correct A1ft: Fully correct ft their v
		A1	Correct value for e ACF
		3	

Q	Answer	Marks	Comments
3(c)	$I = 0.2 \times \frac{4\sqrt{3}}{3} - 0.2 \times (-4\sqrt{3})$ $= \frac{16\sqrt{3}}{15} \text{ [N s]}$	M1 A1	M1: Impulse equation with correct values and any signs A1: Fully correct
		A1	Correct impulse ACF
		3	

	Question 3 Total	9	
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Q	Answer	Marks	Comments
4	At highest point the tension in the string is zero when U is the minimum possible value $m \times 9.8 = \frac{mv^2}{0.8}$ $v^2 = 7.84$ $v = 2.8$ Conservation of energy $\frac{1}{2} \times mU^2 = \frac{1}{2} \times m \times 7.84 + m \times 9.8 \times 1.6$ $U^2 = 39.2$ $U = \frac{14\sqrt{5}}{5} = 6.3 \text{ [to 2sf]}$	M1 A1 M1 A1 A1	Equation for motion at highest point Correct speed at highest point ($v^2 = rg$) M1: Three term equation for conservation of energy A1: Fully correct ($U^2 = 5rg$) Correct value for U AWRT 6.3
	Question 4 Total	5	

Q	Answer	Marks	Comments
5	$m \frac{dv}{dt} = mg \sin 30^\circ - \mu \times mg \cos 30^\circ - mkv$ $\frac{dv}{dt} = \frac{g}{2} - \mu g \frac{\sqrt{3}}{2} - kv$ $\frac{dv}{dt} = \frac{1}{2}(g - \mu g \sqrt{3} - 2kv)$ $\int \frac{1}{(2kv - g + \mu g \sqrt{3})} dv = \int -\frac{1}{2} dt$ $\frac{1}{2k} \ln(2kv - g + \mu g \sqrt{3}) = -\frac{1}{2}t + c$ $v = 0, t = 0 \Rightarrow c = \frac{1}{2k} \ln(\mu g \sqrt{3} - g)$ $\ln(2kv - g + \mu g \sqrt{3}) = -kt + \ln(\mu g \sqrt{3} - g)$ $\frac{2kv - g + \mu g \sqrt{3}}{\mu g \sqrt{3} - g} = e^{-kt}$ $v = \frac{g}{2k}(1 - \mu \sqrt{3})(1 - e^{-kt})$	M1 A1 M1 m1 A1 A1 M1 A1	Four term differential equation from Newton's second law Correct differential equation Uses separation of variables or integrating factor Integrates Correct integration Finds correct constant of integration Solves a logarithmic equation for v Correct expression for v ACF not involving trigonometric functions
	Question 5 Total	8	

Q	Answer	Marks	Comments
6(a)	Let θ be the angle between the string and the vertical at time t $m \times 1.4 \ddot{\theta} = -m \times 9.8 \sin \theta$ $\sin \theta \approx \theta$ $\ddot{\theta} \approx -\frac{9.8}{1.4} \theta = -7\theta$ $\therefore$ SHM As acceleration proportional to the displacement and in the opposite sense	M1 A1 B1 A1 E1	Forms differential equation Correct differential equation Uses small angle approximation Simplifies to SHM form Accept $\ddot{\theta} \approx -\frac{g}{l} \theta$ Concludes that motion is SHM from correct working
		5	

Q	Answer	Marks	Comments
6(b)	Period = $\frac{2\pi}{\sqrt{7}} = \frac{2\sqrt{7}}{7}\pi$ [= 2.3748...] Total angle for 1.8 metres $= \frac{1.8}{1.4} = \frac{9}{7}$ [= 1.286 radians] Angle per complete period $= 4 \times \frac{\pi}{20} = \frac{\pi}{5}$ [= 0.6283... radians] $\left(\frac{9}{7}\right) \div \left(\frac{\pi}{5}\right) = \frac{45}{7\pi}$ [= 2.04627...] [So between 2 and 3 periods required.] $\frac{9}{7} - \frac{2\pi}{5}$ [= 0.029077... radians] $\theta = \frac{\pi}{20} - \left(\frac{9}{7} - \frac{2\pi}{5}\right) = \frac{9\pi}{20} - \frac{9}{7}$ [= 0.12800...] $\frac{9\pi}{20} - \frac{9}{7} = \frac{\pi}{20} \cos(\sqrt{7}t)$ $t = 0.23368...$ seconds Total Time = $2 \times \frac{2\pi}{\sqrt{7}} + 0.23368$ $= 4.98332...$ $= 5.0$ seconds	B1 M1 A1 A1 M1 A1 M1 A1	Correct period stated during working Finds total angle for the motion Obtains exact value or AWRT 2.047 Correct angle for remaining motion since last complete period or correct distance (0.04071...) Finds θ Correct θ Finds time for motion since last period Correct total time AWRT 5.0
		8	
	Question 6 Total	13	

Q	Answer	Marks	Comments
7(a)	$e = \sqrt{5^2 + 3^2} - 5$ $= \sqrt{34} - 5$ $\text{EPE} = \frac{800 \times (\sqrt{34} - 5)^2}{2 \times 5}$ $= 55.238\dots$ $\text{Total EPE} = 110 \text{ [J]}$	M1 A1 M1 A1	Finds extension of strings Correct extension Uses EPE formula with their extension Correct total EPE
		4	

Q	Answer	Marks	Comments
7(b)	Strings slack when height above C is 1 metre Change in GPE = $7 \times 9.8 \times 1$ $= 68.6$ $110.47\dots - 68.6 = \frac{1}{2} \times 7 \times v^2$ $v^2 = 11.971$ $v = 3.4599\dots$ $= 3.5 \text{ [m s}^{-1} \text{ to 2 sf]}$	B1 B1 M1 A1 A1	Finds height when strings at natural length PI Correct change in GPE M1: Three term energy equation A1: Fully correct AWRT 3.5 Allow AWRT 3.4 if 110 J used
		5	

Q	Answer	Marks	Comments
7(c)	$110.5 = 7 \times 9.8h$ $h = 1.6 \text{ [m]} \text{ (to 2 sf)}$	M1 A1	Two term energy equation Correct height
		2	

	Question 7 Total	11	
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Q	Answer	Marks	Comments
8(a)	$0 = 20 \sin 35^\circ t - \frac{1}{2} g \cos 30^\circ t^2$ $t = \frac{40 \sin 35^\circ}{g \cos 30^\circ} \quad [= 2.703...]$ $\dot{x} = 20 \cos 35^\circ - g \sin 30^\circ \times \frac{40 \sin 35^\circ}{g \cos 30^\circ}$ $= 3.1368...$ $\dot{y} = 20 \sin 35^\circ - g \cos 30^\circ \times \frac{40 \sin 35^\circ}{g \cos 30^\circ}$ $= -20 \sin 35^\circ$ $= -11.4715...$ $v = \sqrt{11.4715...^2 + 3.1368...^2}$ $= 11.89...$ $= 12 \text{ [m s}^{-1}\text{]} \text{ (to 2 sf)}$	M1 A1 A1 M1 A1 A1 A1	Equation for motion perpendicular to the plane Correct equation Correct time Finds both components of velocity Correct parallel component Correct perpendicular component Correct speed
		7	

Q	Answer	Marks	Comments
8(b)	Require rebound angle to be greater than 60° $\tan \theta = \frac{11.4715... \times e}{3.1368...}$ $\frac{11.4715... \times e}{3.1368...} > \sqrt{3}$ $0.47 < e < 1$	B1 M1 A1 A1	Identifies 60° Forms expression for tan of the rebound angle Correct inequality Correct range Allow lower limit between 0.46 and 0.48 Accept $0.47 \leq e \leq 1$
		4	

	Question 8 Total	11	
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Q	Answer	Marks	Comments
9(a)	$12 \times 8 \cos 60^\circ = 12v_A + 3v_B$ $16 = 4v_A + 1v_B$ $v_B - v_A = -e(0 - 8 \cos 60^\circ)$ $v_B - v_A = 4e$ $v_A = \frac{16 - 4e}{5}$ $V^2 = (8 \sin 60^\circ)^2 + \left(\frac{16 - 4e}{5}\right)^2$ $V = \sqrt{48 + \left(\frac{16 - 4e}{5}\right)^2}$	M1 M1 A1 A1 M1 A1	Equation for conservation of momentum Equation for coefficient of restitution Two correct equations Correct component of A along line of centres Finds expression for speed of A after the collision Correct expression ACF
		6	

Q	Answer	Marks	Comments
9(b)	$V_{\min} = \sqrt{(8 \sin 60^\circ)^2 + \left(\frac{16 - 4}{5}\right)^2} = \frac{8\sqrt{21}}{5}$ Maximum change in speed = $8 - \frac{8\sqrt{21}}{5} = \frac{40 - 8\sqrt{21}}{5}$	M1 A1	Uses $e = 1$ Correct maximum change in speed ACF
		2	

	Question 9 Total	8	
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